tionality constant (an experimental value) in the correlation function $\overline{u'v'} = \pm \overline{k_1 q^2}$; $\varkappa = 0.011$, turbulence constant in Prandtl equation for jet flows; $\sigma = 12$, similarity coordinate for turbulent mixing. Indices: $\eta$, x, y, t, derivative of the respective function with respect to the coordinates $\eta$, x, y, and t; e, outer boundary of the viscous layer; i = 0, 1, 2, 3, and 4, numbers of the coefficients $a_i$.

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# APPROXIMATE METHOD OF CALCULATING THE TEMPERATURE PROFILE IN A SEMITRANSPARENT MELTING MATERIAL


A. S. Senchenkov



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The integral two-parametric method is used to calculate the temperature profile in a semitransparent melting material; the method takes the specifics of the problem under study into account.


The velocity for the removal of the mass of vitreous heatproof materials under heating action is determined to a significant degree by the temperature profile near the surface, a fact which is related to the strong dependence of the viscosity of these materials on the temperature. Since most vitreous heatproof materials are semitransparent, the temperature profile also determines the amount of heat emitted by the material.

At the same time, as the results of the numerical calculations show [1], the exponential approximations of a transparent film and an opaque film and the approximation of radiant thermal conductivity do not guarantee the satisfactory accuracy in calculating the temperature distribution near the surface if the optic thickness of the liquid film has the order of unity (typical for many heatproof materials).

Below we propose an approximate method for calculating the temperature distribution in a semitransparent material that is applicable for the case given.

The fracture of heatproof materials under heating is described by a system of equations of continuity, motion, energy, and emission transfer with corresponding boundary conditions [2]. We limit ourselves in the present study to the energy equations. We can write the equation for a stationary regime of fracture in dimensionless form [1,2]

$$\frac{d}{dy}\left(\frac{d\theta}{dy} + \theta - f\right) = 0, \tag{1}$$

$$\theta(0) = 1, \qquad \theta(\infty) = \theta_{\mathrm{T}}.$$


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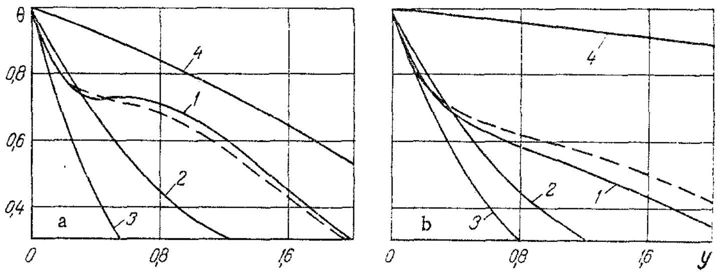


Fig. 1. Temperature profile in material: a, $y_R = 0.2$; b, $y_R = 0.8$. Dashed lines, numerical solution [1], solid lines, approximation solution; 1) biparametric method; 2) opaque film [1]; 3) transparent film [1]; 4) radiant thermal conductivity.

By integrating Eq. (1) within the limits from y to ∞ and disregarding the emission of the cold regions of the material, we obtain

$$\frac{d\theta}{dy} + \theta - \theta_{\text{т}} - f = 0. \tag{2}$$

The quantity of the density of the emission flow is determined by the temperature profile and by the thermooptic properties of the material. For a gray body, f is a function of the integral containing the fourth degree of temperature [1, 3]. Thus, Eq. (2) is a nonlinear integrodifferential equation.

The approximate solution of energy equation (1) is found by the integral method. The temperature distribution is approximated by a two-parametric family of functions:

$$\theta = \theta(y,\ A,\ B). \tag{3}$$

We must write two conditions to determine the unknown parameters A and B. One of these follows from the law of energy conservation,

$$\frac{d\theta}{dy}(0,\ A,\ B) + 1 - \theta_{\text{т}} - f(0,\ A,\ B) = 0. \tag{4}$$

The second condition is usually determined by the integration of Eq. (1) with any weighting function. The method leads, on the average, to a sufficient approximation of the temperature profile along the whole thickness of the material. However, significant deviations from the true temperature distribution are possible in separate segments. A characteristic of the problem is the necessity of a good approximation of the temperature profile near the surface in the layer with the optic thickness of the order of unity, since this part of the profile determines the quantity of heat emitted by the material and the velocity of the removal of the material in the fluid phase.

Thus, we assume that as a second condition for determining parameters A and B we can use the condition of realizing, "on the average," Eq. (1) in the surface layer of the material with the optic thickness equal to unity. By integrating Eq. (1) within the limits from zero to $y_R$ ($y_R$ is the mean free path of the emission), with (4) taken into account, we obtain

$$\frac{d\theta}{dy}(y_R,\ A,\ B) + \theta(y_R,\ A,\ B) - \theta_{\text{т}} - f(y_R,\ A,\ B) = 0. \tag{5}$$

Thus, the integral method for (1) reduces to the collocation method for Eq. (2). The points of the collocation are points $y = 0$ and $y = y_R$.

After solving (4) and (5) together, we find the values for the parameters A and B.

We compare the results of the suggested method with the results of the numerical calculations [1]. The density of the emission flow is calculated according to the equation in [1]:

$$f(y) = \frac{\bar{f}}{y_R} \int_0^\infty \left[ \exp\left(-\frac{|y - y'|}{y_R}\right) \operatorname{sign}(y - y') + R_{eff} \exp\left(-\frac{y + y'}{y_R}\right) \right] \theta^4(y')\, dy', \tag{6}$$

where $\bar{f}$ is the density of the flow of self-radiation at constant temperature in the material ($\theta \equiv 1$) and in the absence of reflection; $R_{eff}$ is the effective reflectivity of the surface of the material from a side of the body.

The temperature profile is approximated by the dependence

$$\theta = (1 + Ay^2)\exp(-By). \tag{7}$$

In Fig. 1 we provide the comparison of the temperature distribution in heatproof materials which is obtained by various approximate methods and the distribution obtained by numerical methods. Curves a correspond to the case when the optic thickness of the film has the order of unity, and curves b to the case when the optic thickness of the film is considerably less than unity. As we see, in both cases the biparametric method reflects the character of the temperature variation in heatproof materials very well.

The errors in calculating both the velocity of the removal of mass in the liquid phase and the degree of blackness in the material are determined according to the results obtained. The error for calculating the velocity does not exceed 10% in the cases studied, while the approximation of the opaque film exceeds this quantity two times. The error in approximating the transparent film depends considerably on the optic thickness of the film and increases from 10% for $y_R = 0.8$ to 60% for $y_R = 0.2$. The approximation of the radiant thermal conductivity yields the greatest error; the velocity of the removal of mass is exceeded by an order.

Concerning the degree of blackness, the error of the approximate methods in the cases studied lies within these limits: the biparametric method and the approximation of the opaque film is from 3 to 10%, and the approximation of the transparent film is from 30 to 35%.

Thus, the integral biparametric method is accurate enough in practice for calculating both the removal of mass in the liquid phase as well as the degree of blackness in cases when the optic thickness of the liquid film has an order of unity or less than unity.

## NOTATION

$\theta$, dimensionless temperature; $\theta_T$, dimensionless temperature inside body; y, dimensionless coordinate along normal to body; $y_R$, dimensionless free path length of emission; f, density of emission flow.

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